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MOTION OF A GAS BEHIND A PLANE DETONATION WAVE ORTHOGONAL TO THE FREE SURFACE

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The problem of detonation of one quarter of a space filled with explosive and initiated on one of the faces is examined. The finding of the solution in the perturbed region is reduced to the solution of Goursat's problem for a quasi-linear differential equation of second order with two independent variables. This problem is solved by the numerical method of characteristics. An examination of singular points is presented. The solution in the perturbed region and the form of the free surface are obtained.

The problem of gas motion behind an expanding detonation wave in a space with a conical cutout was examined in papers [1, 2].

1. Let us examine the infinite region

$$x_1 > 0, \qquad x_2 < 0 \quad \text{for} \quad t < 0 \tag{1.1}$$

filled with immovable explosive of constant density ρ_0. The pressure p in the entire space is equal to zero.

It will be assumed that the products of explosion are described by the following equation of state

$$p = \gamma\rho^\gamma, \qquad \gamma > 1 \qquad (\rho_0 = \gamma/(\gamma+1)) \tag{1.2}$$

At the instant of time $t = 0$ the explosive is initiated on the surface $x_1 = 0$. A plane normal detonation wave, which is orthogonal to the free surface $x_2 = 0$ propagates with the constant velocity $D = \gamma + 1$ from the plane of initiation $x_1 = 0$

2. For $t > 0$ the motion is self-similar with the following independent variables

$$\xi_1 = x_1/t, \qquad \xi_2 = x_2/t \tag{2.1}$$

The straight line $\xi_1 = D$ corresponds to the front of the detonation wave. Behind the wave front the gasdynamic parameters assume the following values

$$u_1 = 1, \quad u_2 = 0, \quad \rho = 1, \quad c = \gamma \tag{2.2}$$

where u_1 and u_2 are components of the velocity vector, c is the speed of sound. Far from the straight line $\xi_2 = 0$ in the region

$$-D/(\gamma - 1) < \xi_1 < D$$

the motion is one-dimensional

$$u_1 = \frac{2}{\gamma+1}\left(\xi_1 - \frac{D}{2}\right), \qquad u_2 = 0, \qquad c = \frac{\gamma-1}{2} u_1 + \frac{D}{2} \tag{2.3}$$

The straight line $\xi_1 = -D / (\gamma - 1)$ corresponds to the plane front of exhaust into vacuum.

In the vicinity of the line $\xi_2 = 0$ the one-dimensional motion (2.3) is perturbed by a centered wave of expansion (a Prandtl-Meyer wave) emanating from the point A ($\xi_1 = D$, $\xi_2 = 0$). The region of perturbed flow (1) is separated from the region of one-dimensional flow (0) by a weak discontinuity passing through the points A and B ($\xi_1 = -D / (\gamma - 1)$, $\xi_2 = 0$). This discontinuity can be found by solving the differential equation for the characteristics of one-dimensional flow (2.3)

$$\frac{d\xi_2}{d\xi_1} = \frac{\xi_2^2 - c^2}{2\xi_2 c} \tag{2.4}$$

The function c (ξ_1) must be taken from (2.3)

$$c = \mu^2 \xi_1 + \frac{D}{\gamma+1}, \qquad \mu = \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2} \tag{2.5}$$

Integrating (2.4) we find the boundary characteristic for $\gamma \neq 3$

$$\xi_2 = -\frac{c}{\mu}\left\{\frac{1}{\tau}\left[1 - \left(\frac{c}{\gamma}\right)^{\tau}\right]\right\}^{1/2}, \qquad \tau = \frac{3-\gamma}{\gamma-1} \tag{2.6}$$

In Eq. (2.6) the "minus" sign is taken because it must be that $\xi_2 \leqslant 0$. The constant in (2.6) is selected from the condition $\xi_2 = 0$ for $\xi_1 = D$. The function c (ξ_1) is determined from (2.5). From Eq. (2.6) it is evident that the boundary characteristic passes through the point B, where $c = 0$. For $\gamma = 3$ we obtain instead of (2.6)

$$\xi_2 = -c\left(-2\ln\frac{c}{3}\right)^{1/2} \tag{2.7}$$

Equation (2.7) can be obtained from (2.6) by passing to the limit for $\gamma \to 3$, therefore the case $\gamma = 3$ is subsequently not treated separately and it is assumed that the passage to the limit is possible.

In the point A the boundary characteristic has an infinite slope, i.e. it is tangent to the detonation wave. In the point B the slope depends on γ. For $\gamma < 3$ in the point B

$$d\xi_2/d\xi_1 = -\mu/\sqrt{\tau} \tag{2.8}$$

For $\gamma \geqslant 3$ the boundary characteristic has infinite slope, consequently it is tangent to the boundary with the vacuum $\xi_1 = -D / (\gamma - 1)$.

In the point M the function (2.6) has a minimum, here

$$\xi_1 = -\frac{D}{\gamma-1} + 2\gamma\,(2\mu^2)^{1/\tau - 1}, \qquad \xi_2 = -\gamma\,(2\mu^2)^{1/\tau} \tag{2.9}$$

In the point A region (1) adjoins the Prandtl-Meyer expansion wave

$$\begin{aligned} u_1 &= D - \gamma a \cos\alpha + \gamma b \cos\left(\frac{a}{b}\alpha\right), & a &= \frac{1}{2}\left(\frac{1}{\mu} + 1\right) \\ u_2 &= \gamma a \sin\alpha - \gamma b \sin\left(\frac{a}{b}\alpha\right), & b &= \frac{1}{2}\left(\frac{1}{\mu} - 1\right) \\ c &= \gamma \cos\left(\frac{\alpha}{2b}\right) & & (0 \leqslant \alpha \leqslant \pi b) \end{aligned} \tag{2.10}$$

Here α is a parameter. For $\alpha = 0$ we obtain from equations (2.10) the values of quantities behind the front of the detonation wave (2.2). For $\alpha = \pi b$ the speed of sound becomes zero.

The perturbed region (1) is closed by the curvilinear front of outflow into vacuum. This front passes through the points A and B.

3. The solution in the perturbed region is obtained as a result of interaction of two simple waves (2.3) and (2.10), therefore it represents a double wave [3]. Simple and double waves are generalizations of Riemann waves to the multidimensional case. The function $c = c(u_1, u_2)$, which connects the speed of sound with the speed of the material, satisfies the quasi-linear differential equation of the second order

$$a_{22}(\varphi_{11} + 1) - 2a_{12}\varphi_{12} + a_{11}(\varphi_{22} + 1) = 0 \tag{3.1}$$

Here

$$\varphi = \frac{c^2}{\gamma - 1}, \qquad \varphi_i = \frac{\partial \varphi}{\partial u_i}, \qquad \varphi_{ij} = \frac{\partial^2 \varphi}{\partial u_i \partial u_j} \tag{3.2}$$

$$a_{ij} = \varphi_i \varphi_j - c^2 \delta_{ij} \qquad (i, j = 1, 2)$$

If the function $c(u_1, u_2)$ is determined from Eq. (3.1), then the solution in the plane of self-similar variables $\xi_1 \xi_2$ is found from the following relationship:

$$\xi_i = u_i + \varphi_i \qquad (i = 1, 2) \tag{3.3}$$

In Eq. (3.1) we make a substitution of variables

$$x = \frac{k}{\gamma}(1 - u_1), \qquad y = \frac{k}{\gamma} u_2, \qquad z = \frac{c}{\gamma} \qquad \left(k = \frac{\gamma - 1}{2}\right) \tag{3.4}$$

Computing the derivatives of function $\varphi(u_1, u_2)$ from (3.4) and substituting them into (3.1), we obtain the equation

$$(z_y^2 - 1) z_{xx} - 2 z_x z_y z_{xy} + (z_x^2 - 1) z_{yy} = \frac{1}{z}[2 + \tau(2 - z_x^2 - z_y^2)] \tag{3.5}$$

Instead of relationships (3.3) we have

$$\xi_1 = 1 - \gamma\left(\frac{x}{k} + z z_x\right), \qquad \xi_2 = \gamma\left(\frac{y}{k} + z z_y\right) \tag{3.6}$$

Conditions of contiguity of the double wave to the one-dimensional flow in region (0) give

$$y = \theta, \qquad z = 1 - x, \qquad z_x = -1, \qquad z_y = -\theta(x) \tag{3.7}$$

$$\theta(x) = \frac{1}{\mu}\left\{\frac{1}{\tau}[1 - (1 - x)^\tau]\right\}^{1/2} \tag{3.8}$$

From the condition of contiguity to the centered simple wave (2.10) we obtain

$$x = k\left[a \cos\alpha - b\cos\left(\frac{a}{b}\alpha\right) - 1\right] \tag{3.9}$$

$$y = k\left[a \sin\alpha - b \sin\left(\frac{a}{b}x\right)\right] \qquad (0 \leqslant x \leqslant \pi b)$$

$$z = \cos\left(\frac{\alpha}{2b}\right), \qquad z_x = -\frac{1}{z}\left(1 + \frac{x}{k}\right), \qquad z_y = -\frac{y}{kz}$$

Derivatives z_x and z_y in (3.9) are determined from Eqs. (3.6) after substitution of values $\xi_1 = \gamma + 1$, $\xi_2 = 0$. The problem of finding the solution in the perturbed region is reduced to the solution of Eq. (3.5) with conditions on characteristics (3.7), (3.9). The discriminant of equation (3.5)

$$\delta = z_x^2 + z_y^2 - 1 \tag{3.10}$$

is everywhere on determinants (3.7), (3.9) greater than zero, with the exception of the point $x = y = 0$, where $z_x = -1$, $z_y = 0$,i.e. $\delta = 0$. In this point the characteristics which carry the initial data are tangent with each other.

4. For determination of the function $z\ (x,\ y)$ the problem of Goursat is obtained. The region of integration in the xy -plane represents a curvilinear triangle $A°$, $B°$, $C°$, the vertices of which have the coordinates

$$x = 0,\quad y = 0, \qquad x = 1,\quad y = 0, \qquad x = x_0,\quad y = y_0$$

The values x_0, y_0 corresponding to the point $C°$, are computed from Eqs. (3.9) for $\alpha = \pi b$

$$x_0 = \frac{km}{\mu} - k, \qquad y_0 = -\frac{kn}{\mu} \qquad \left(m = \sin\frac{\pi}{2\mu},\quad n = \cos\frac{\pi}{2\mu}\right) \tag{4.1}$$

In the following it will be assumed that $\gamma > 1.25$, then $n < 0$. In the vicinity of point $A°$ on characteristics (3.8) and (3.9) the following expansions are applicable:

$$y = 0, \quad z = 1 - x, \quad z_x = -1, \quad z_y = -\frac{1}{\mu} x^{1/2} \tag{4.2}$$

$$y = \lambda_1 x^{3/2},\ z = 1 - x - \frac{x^2}{2\mu^2}, \quad z_x = -1 - \frac{x}{\mu^2}$$

$$z_y = -\frac{\lambda_1}{k} x^{3/2}, \qquad \lambda_1 = \frac{a_1}{2\mu}, \quad a_1 = \frac{4\sqrt{2}}{3} \tag{4.3}$$

Let us examine the behavior of the solution of the Goursat problem in the vicinity of the point $A°$. We assume

$$\lambda = \lambda_1^{-1} y x^{-3/2} \tag{4.4}$$

and look for a solution in the form

$$z = 1 - x - \frac{x^2}{2\mu^2} \psi\ (\lambda) \tag{4.5}$$

The values of the parameter $\lambda = 0$ and $\lambda = 1$ correspond to characteristics (4.2) and (4.3). Differentiating (4.5) with respect to x and y, we find

$$z_x = -1 - \frac{x}{\mu^2} (\psi - {}^3/_4\lambda\psi'), \qquad z_y = -\frac{x^{1/2}\psi'}{a_1\mu} \tag{4.6}$$

Substituting (4.5) and (4.6) into (3.5), (4.2) and (4.3) we obtain for the function $\psi\ (\lambda)$ the following differential equation:

$$\psi''\ (\psi - \lambda^2) - {}^1/_4\psi'^2 + \lambda\psi' - {}^8/_9\ (\psi - 1) = 0 \tag{4.7}$$

and the boundary conditions

$$\psi\ (0) = 0, \quad \psi'\ (0) = a_1, \qquad \psi\ (1) = 1, \quad \psi'\ (1) = 0 \tag{4.8}$$

The expansion of the solution of Eq. (4.7) in the vicinity of the singular point $\lambda = 0$ has the form

$$\psi = a_1\lambda - a_2\lambda^{3/2} - a_3\lambda^2 - a_4\lambda^{5/2} - \ldots \tag{4.9}$$

$$a_3 = {}^1/_9\ (1 + 6a_2^2 / a_1^3), \qquad a_4 = {}^1/_{18}\ (1 + 9a_3)\ a_2 / a_1, \ldots \tag{4.10}$$

Coefficients of the series (4.9) depend on the arbitrary parameter $a_2 > 0$, which is determined in the numerical integration of Eq. (4.7) from the condition that the solution passes through the second singular point $\lambda = 1$. As a result of numerical integration we obtained $a_2 = 0.6105$. For small λ we obtain from (4.4), (4.5) and (4.9) the following expansion

$$z = 1 - x - \mu^{-1} x^{1/2} y + \frac{1}{3\mu^{1/2}} x^{-1/4} y^{3/2} \tag{4.11}$$

In the vicinity of the point $\lambda = 1$ the expansion for ψ has the form

$$\psi = 1 - 0.84\ (1 - \lambda)^{3/2} \tag{4.12}$$

In the vicinity of the characteristic $y = 0$ we shall seek a solution in the form of an expansion in powers of y

$$z = 1 - x - y\theta(x) + y^{3/2} F(x) \tag{4.13}$$

The expansion (4.11) represents the limiting case for expansion (4.13) when x tends to zero. Therefore we must have for $F(x)$

$$\lim_{x\to 0} F(x)\, x^{1/4} = \frac{1}{3\mu^{1/2}} \tag{4.14}$$

We substitute (4.13) into (3.5) for $F(x)$ and obtain the equation

$$\frac{F'}{F} = -\frac{1}{2}\frac{\theta'}{\theta} - \frac{\tau}{1-x} \tag{4.15}$$

Integrating (4.15), we find

$$F(x) = \frac{1}{3\mu}\theta^{-1/2}(1-x)^{\tau} \tag{4.16}$$

The constant of integration in (4.16) was determined from the relationship (4.14). For $z = 0$ we obtain from (4.13) the form of the boundary with the vacuum in the vicinity of the point B°:

for $\gamma < 3$
$$y = \mu\sqrt{\tau}\,(1-x) \tag{4.17}$$

for $\gamma = 3$
$$y = (1-x)\ [-2\ \ln(1-x)]^{-1/2} \tag{4.18}$$

for $\gamma > 3,\ -1 < \tau < 0$
$$y = \mu\sqrt{-\tau}\,(1-x)^{1-1/2\tau} \tag{4.19}$$

For $\gamma \geqslant 3$ the boundary with vacuum is tangent to the characteristic $y = 0$ and in the point B° it has infinite curvature. In the vicinity of point B° the quantities zz_x and zz_y / y, computed from (4.13), tend to zero, so that we obtain from (3.6)

$$x = k(1 - \xi_1)/\gamma, \qquad y = k\xi_2/\gamma \tag{4.20}$$

Substituting (4.20) into (4.17) - (4.19), we obtain the asymptotics of the free boundary in the vicinity of the point B.

5. Let us examine the behavior of the solution of the problem (3.5), (3.7), (3.9) in the vicinity of the point C°. Eliminating α from (3.9) we obtain

$$z = k^{-1/2}[k(k+1) - (k+x)^2 - y^2]^{1/2} \tag{5.1}$$

For (3.9) in the vicinity of the point C° we can write the expansions

$$y-y_0=\frac{m}{n}(x-x_0)+\frac{(x-x_0)^2}{2\mu n^3},\qquad x>x_0 \tag{5.2}$$

$$z=-\frac{1}{n}(x-x_0),\quad z_x=\frac{mn}{\mu(x-x_0)},\quad z_y=-\frac{n^2}{\mu(x-x_0)} \tag{5.3}$$

The expansion for z is obtained from (5.1) taking into account terms of second order in smallness in (5.2).

We shall seek solutions of Eq. (3.5) in the vicinity of point C° in the form

$$z=-\frac{1}{n}(x-x_0)f(\zeta) \tag{5.4}$$

$$\zeta=2\mu mn^2\left[\frac{n}{m}\frac{(y-y_0)}{(x-x_0)^2}-\frac{1}{x-x_0}\right] \tag{5.5}$$

For the expansion (5.2) the quantity $\zeta=1$. For derivatives of z with respect to x and y we have

$$z_x=-\frac{1}{n}(f-\zeta f')+\frac{2\mu mnf'}{x-x_0},\qquad z_y=-\frac{2\mu n^2f'}{x-x_0} \tag{5.6}$$

Substituting the function z from (5.4) into Eq. (3.5), we obtain a differential equation of second order for the function f

$$(1+f^2)f''+2f'(f-\zeta f')+\frac{\tau f'}{f}=0 \tag{5.7}$$

From Eqs. (5.3) we find the boundary conditions

$$f(1)=1,\qquad f'(1)=1+{}^1/_2\tau \tag{5.8}$$

We always have the value $f'(1)>0$ because $\tau>-1$. The function $f(\zeta)$ must be such that for some $\zeta=\zeta_0<1$ it becomes zero. In the vicinity of point $\zeta=1$ the following expansion is valid for the function $f(\zeta)$

$$f=1+(1+{}^1/_2\tau)(\zeta-1) \tag{5.9}$$

For $\tau=0$ this gives the exact solution for the problem (5.7), (5.8). For small τ Eq. (5.9) can be regarded as an approximate solution. In this case the value ζ_0 is approximately equal to 0.5τ, for $\tau=0$, $\zeta_0=0$.

From relationships (3.6) we find ξ_1 and ξ_2 for the solution (5.4) when $x=x_0$

$$\xi_1=D-\frac{\gamma m}{\mu}(1-2\mu^2ff'),\qquad \xi_2=-\frac{\gamma n}{\mu}(1-2\mu^2ff') \tag{5.10}$$

Eliminating ff' from (5.10), we obtain

$$\xi_2=(\xi_1-D)\,n\,/\,m \tag{5.11}$$

It follows from this that the point C° in the plane $\xi_1\xi_2$ transforms into a section of straight line (5.11) which is confined between points A and C. The coordinates of point C are computed from (5.10) by substituting $\zeta=\zeta_0$. The neighborhood of point C° transforms into the neighborhood of the straight line (5.11). For single-valued behavior of representation (5.10) it is necessary that the quantity ff' be a monotonic function of ζ in the interval $(1,\zeta_0)$.

If we assume in (5.7) and (5.8) that

$$g={}^1/_2f^2,\qquad g'=ff' \tag{5.12}$$

then we obtain the following problem for function $g(\zeta)$:

$$g'' = \frac{g'^2}{g(1+2g)}(\zeta g' - g - \varkappa) \qquad \left(\varkappa = \frac{\tau-1}{2} = \frac{2-\gamma}{\gamma-1}\right) \tag{5.13}$$

$$g(1) = {}^1/_2, \qquad g'(1) = 1 + {}^1/_2\tau$$

From (5.13) it follows that we have for $\zeta = 1$

$$g' > 0, \qquad g'' > 0 \tag{5.14}$$

It will be shown that conditions (5.14) are satisfied everywhere in the interval $(1, \zeta_0)$. The solution of problem (5.13) can be found in quadratures

$$\frac{1}{g'} = \frac{1}{1+{}^1/_2\tau} + I \tag{5.15}$$

$$\zeta = (g+\varkappa)\frac{1}{g'} + \Phi \tag{5.16}$$

$$I(\varkappa, g) = \int\limits_g^{1/2} \frac{\Phi dg}{g(1+2g)} \tag{5.17}$$

$$\Phi(\varkappa, g) = 2^{\varkappa+1}\mu^2 g^\varkappa \left(\frac{1}{2} + g\right)^{1/2-\varkappa} \tag{5.18}$$

The function $\Phi(\varkappa, g) > 0$ for all $g > 0$, therefore $I(\varkappa, g) \geqslant 0$. We substitute (5.16) into (5.13) and obtain

$$g'' = \frac{\Phi g'^3}{g(1+2g)} \tag{5.19}$$

It follows from (5.15) and (5.19) that conditions (5.14) are valid everywhere in the interval $(1, \xi_0)$. This indicates that f and ff' are monotonic functions.

We substitute (5.15) into (5.16) and obtain the equation (5.20)

$$\zeta(\varkappa, g) = \Phi + (g + \varkappa)\left(I + \frac{1}{1+{}^1/_2\tau}\right) \tag{5.20}$$

The integral (5.17) is taken in elementary functions only for

$$\gamma = 3, \qquad \gamma = 1 + 1/(N+1), \qquad N = 0, 1, 2, \ldots \tag{5.21}$$

For example, for $\varkappa = 0 \quad (\gamma = 2)$

$$\zeta(0, g) = \frac{\sqrt{2}}{3}\left[r + g\left(\sqrt{2} + \ln\frac{r+1}{\sqrt{2}+1} - \ln\frac{r-1}{\sqrt{2}-1}\right)\right] \tag{5.22}$$

$$r = (1+2g)^{1/2}$$

From here $\zeta_0 = \zeta(0, 0) = \sqrt{2}/3$. Integrating I by parts and taking $g = 0$ in (5.20), we obtain for arbitrary $\varkappa$

$$\zeta_0 = \tau\mu^2(1 + {}^1/_2 R), \qquad R = \int\limits_0^{1/2}\left(\frac{2g}{{}^1/_2 + g}\right)^\varkappa \frac{dg}{({}^1/_2 + g)^{3/2}} \tag{5.23}$$

Differentiating ζ_0 with respect to $\varkappa$, we find

$$\frac{d\zeta_0}{d\varkappa} = 4\mu^4\left(1 + \frac{1}{2}R\right) + \frac{1}{2}\tau\mu^2\frac{dR}{d\varkappa} \tag{5.24}$$

Setting $s = 2g/({}^1/_2 + g)$ in (5.23), we obtain the estimates

$$\frac{1}{\sqrt{2}(\varkappa+1)} \leqslant R \leqslant \frac{1}{\varkappa+1} \tag{5.25}$$

$$-\frac{1}{(\varkappa+1)^2} \leqslant \frac{dR}{d\varkappa} \leqslant -\frac{1}{\sqrt{2}(\varkappa+1)^2} \tag{5.26}$$

Utilizing (5.25) and (5.26), we find from (5.24) that $d\zeta_0/d\varkappa > 0$, i.e. $\zeta_0(\varkappa)$ is a monotonically increasing function. From (5.25) we can obtain an approximate equation for the computation of ζ_0 (for $\varkappa > -0.6$ the error is less than 2%)

$$\zeta_0 = \tau\mu^2\left(1 + \frac{0.4}{\varkappa+1}\right) \tag{5.27}$$

We can show that $\zeta_0 \to -\infty$ logarithmically for $\varkappa \to -1$.

The curve $z = 0$ in the vicinity of the point C° has the form

$$y - y_0 = \frac{m}{n}(x - x_0) + \frac{\zeta_0}{2\mu n^3}(x - x_0)^2 \tag{5.28}$$

i.e. it is tangent to the characteristic (5.2). For $\gamma = 3$, $\zeta_0 = 0$ the line $z = 0$ in the vicinity of the point C° is also a straight line. For $\gamma \neq 3$ it has a curvature different from zero. For $\gamma < 3$ the curve $z = 0$ lies in the region between the characteristic (5.2) and the tangent to the characteristic in the point C°. For $\gamma > 3$ the curve is outside this region.

From (5.10) we calculate the coordinates of point C

$$\xi_{10} = D + \frac{m}{n}\xi_{20}, \qquad \xi_{20} = -\frac{\gamma n}{\mu}[1 - 2\mu^2 g'(\varkappa, 0)] \tag{5.29}$$

For $\varkappa > 0$ the derivative $g'(\varkappa, 0) = \varkappa / \zeta_0$. For $\varkappa \leqslant 0$ it is equal to zero. Utilizing Eqs. (3.6), we find that the line (5.28) in the plane $\xi_1\xi_2$ transforms into the curve

$$\xi_2 - \xi_{20} = -\frac{m}{n}(\xi_1 - \xi_{10}) + \frac{k\zeta_0}{2\gamma\mu n^3}(\xi_1 - \xi_{10})^2 \tag{5.30}$$

In this manner the line for the level $z = 0$ in the vicinity of the point C consists of two pieces of orthogonal curves (5.11) and (5.30).

6. The problem of Goursat for the Eq. (3.5) was solved by the numerical method of characteristics. In all node points of the region $A^\circ B^\circ C^\circ$ the values of x, y, z, z_x and z_y were computed. The calculation was started from the point A°. In the first calculation point the values z, z_x and z_y were computed from Eqs. (4.5) and (4.6). Through this point a characteristic of the same family as the characteristic $A^\circ B^\circ$ was drawn. For small y this characteristic has the form

$$y = \theta_0 / \theta \qquad (\theta_0 = \text{const}) \tag{6.1}$$

For computations on the characteristic (6.1) the expansion (4.13) was used. All subsequent points were computed by the method of characteristics. In the vicinity of the point C° the method of characteristics gave low accuracy. Here the solution was obtained with the aid of expansion (5.4).

As an example we present the results for $\gamma = 3$. In this case $z = \rho$ is the density of the gas

$$m = 0.796, \quad n = -0.606, \quad x_0 = 0.125, \quad y_0 = 0.857, \quad \xi_{10} = 0.624, \quad \xi_{20} = 2.57 \tag{6.2}$$

The line for the level $\rho = 0$ in the vicinity of the point C° is the straight line

$$y - y_0 = 1.314\,(x - x_0) \qquad (6.3)$$

The region of integration in the xy plane is represented in Fig. 1. The transformation

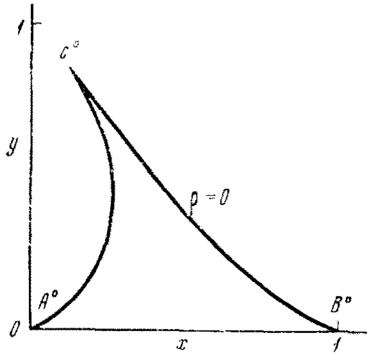

Fig. 1.

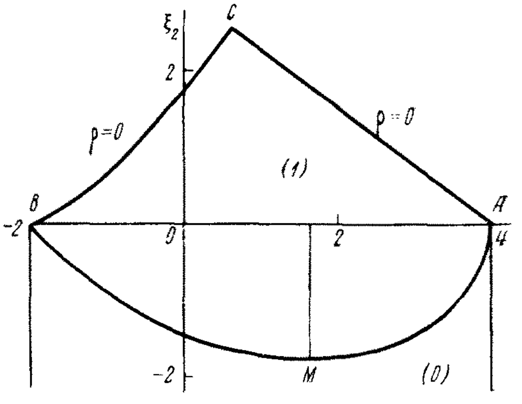

Fig. 2.

to the $\xi_1\xi_2$ plane is accomplished with the aid of Eqs. (3.6). The flow picture in the plane of self-similar coordinates is shown in Fig. 2. The line for the level $\rho = 0$ in the vicinity of the point C consists of two pieces of orthogonal straight lines

$$\xi_2 = -0.761\,(\xi_1 - 4), \qquad \xi_2 - \xi_{20} = 1.314\,(\xi_1 - \xi_{10}) \qquad (6.4)$$

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